Calculus Warm Up #3- 1

$f(x) = |x + 3|$

1. Find the derivative at $c = -3$,

using: $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

HW Questions: p. 103

In Exercises 5–14, use the definition of the derivative to find $f'(x)$.

7. $f(x) = -5x$

11. $f(x) = \frac{1}{x - 1}$

$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x-1)} - \frac{1}{(x-1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{x-1-(x+\Delta x-1)}{(x-1)(x+\Delta x-1)}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\cancel{\Delta x}} \cdot \frac{-\cancel{\Delta x}}{(x-1)(x+\Delta x-1)}$$

$$= \frac{-1}{(x-1)(x-1)}$$

$$f'(x) = \frac{-1}{(x-1)^2}$$

13) f(t) = t^3 - 12t

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(t) = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^3 - 12(t+\Delta t) - (t^3 - 12t)}{\Delta t}$$

$$f'(t) = \lim_{\Delta t \to 0} \frac{\cancel{t^3} + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 \cancel{- 12t} - 12\Delta t \cancel{- t^3} \cancel{+ 12t}}{\Delta t}$$

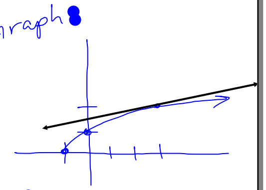
$$f'(t) = \lim_{\Delta t \to 0} \frac{\cancel{\Delta t}(3t^2 + 3t\Delta t + (\Delta t)^2 - 12)}{\cancel{\Delta t}}$$

$$f'(t) = 3t^2 - 12$$

In Exercises 15–20, find the equation of the tangent line to the graph of *f* at the indicated point. Then verify your answer by sketching both the graph of *f* and the tangent line.

Function	*Point of Tangency*
15. $f(x) = x^2 + 1$	(2, 5)
19. $f(x) = \sqrt{x+1}$	(3, 2)

$\rightarrow$ Graph:



$$m = f'(3) = \lim_{x \to 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3}$$

$$f'(3) = \lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$$

$$f'(3) = \lim_{x \to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$f'(3) = \lim_{x \to 3} \frac{1}{\sqrt{x+1}+2}$$

$$f'(3) = \frac{1}{4} \rightarrow \text{slope at } x = 3$$

$\rightarrow$ tangent line:

$$y - 2 = \frac{1}{4}(x-3)$$

In Exercises 21–26, use the alternate form of the derivative (Theorem 3.1) to find the derivative at $x = c$ (if it exists).

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

23. $f(x) = x^3 + 2x^2 + 1,\ c = -2$

$$f'(-2) = \lim_{x \to -2} \frac{x^3 + 2x^2 + 1 - (-8 + 8 + 1)}{x + 2}$$

$$= \lim_{x \to -2} \frac{x^3 + 2x^2}{x + 2}$$

$$= \lim_{x \to -2} \frac{x^2(x+2)}{(x+2)}$$

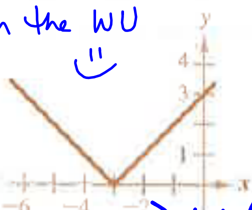
$$= (-2)^2$$

$$= 4$$

In Exercises 27–36, find every point at which the function is differentiable.

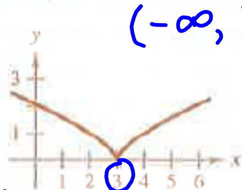
27. $f(x) = |x + 3|$

from the WU :)



$(-\infty, -3) \cup (-3, \infty)$

31. $f(x) = (x - 3)^{2/3}$



$(-\infty, 3) \cup (3, \infty)$

$\frac{k}{0}$

vertical tangent @ $x = 3$

35. $f(x) = \begin{cases} 4 - x^2, & 0 < x \\ x^2 - 4, & x \leq 0 \end{cases}$

$(-\infty, 0) \cup (0, \infty)$

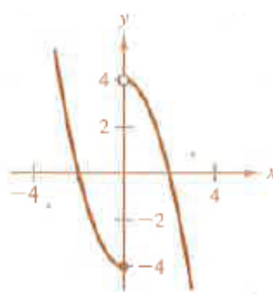


FIGURE FOR 35

In Exercises 39–42, find the derivatives from the left and from the right at $x = 1$ (if they exist). Is the function differentiable at $x = 1$?

39. $f(x) = \sqrt{1 - x^2}$

In Exercises 43 and 44, find an equation of the line that is tangent to the graph of *f* and parallel to the given line.

Function	Line
43. $f(x) = x^3$	$3x - y + 1 = 0$

$\rightarrow \quad y = 3x + 1$

$m = 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \cancel{- x^3}\right)$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left(3x^2 + 3x\Delta x + (\Delta x)^2\right)}{\cancel{\Delta x}}$$

$$f'(x) = 3x^2$$

$\rightarrow \quad 3 = 3x^2$

$x = \pm 1$

on $f(x)$

$(-1, -1) \quad (1, 1)$

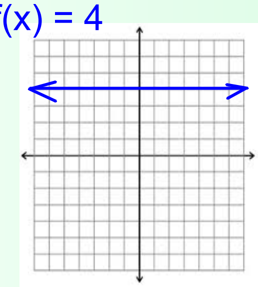
Notes: 3.3 Differentiation Rules

Constant Rule:

The derivative of a constant is zero

$\frac{d}{dx}[c] = 0$, c is a real number

f(x) = 4



1. Find $f'(x)$ if $f(x) = 4$

Power Rule:

If n is a rational number, then

$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Ex:

$f(x) = x^2$

$f'(x) = 2x^{2-1}$

$f'(x) = 2x$

1. Find $f'(x)$ if $f(x) = x^3$

$f'(x) = 3x^{3-1}$

$f'(x) = 3x^2$

2. Find $f'(x)$ if $f(x) = \frac{1}{x^2}$

Rewrite: $f(x) = x^{-2} \rightarrow f'(x) = -2x^{-2-1}$

$f'(x) = -2x^{-3}$

$f'(x) = -\frac{2}{x^3}$

Constant Multiple Rule:

If *f* is a differentiable function and *c* is a real number, then

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

1. Find $\frac{dy}{dx}$ if $y = \frac{2}{x}$

Rewrite:
$y = 2x^{-1}$

$y' = 2(-1)x^{-1-1}$

$y' = -\frac{2}{x^2}$

2. Find y' if $y = \frac{4t^2}{5}$

$y' = \frac{4}{5} \cdot 2t^{2-1}$

$y' = \frac{8t}{5}$

Sum and Difference Rules:

The derivative of the sum (or difference) of two differentiable functions is the sum (or difference) of their derivatives.

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$\frac{d}{dx}[5] = 0$

1. Find $f'(x)$ if $f(x) = x^3 - 4x + 5$

$f'(x) = 3x^2 - 4$

2. Find $\frac{dy}{dt}$ if $y = -\frac{t^4}{2} + 3t^3 - 2t$

$y' = -\frac{1}{2}(4t^3) + 3(3t^2) - 2$

$y' = -2t^3 + 9t^2 - 2$

Find the derivatives of the following functions.

Rewrite | y'

1. $y = \dfrac{5}{2x^3}$ $\quad y = \frac{5}{2}x^{-3}$ $\quad y' = \frac{5}{2}(-3)x^{-3-1}$

$y' = -\frac{15}{2x^4}$

2. $y = \dfrac{7}{(2x)^{-2}}$ $\quad y = 7(2x)^2$ $\quad y' = 28(2)x^1$

$y = 28x^2$ $\quad y' = 56x$

3. $y = \dfrac{1}{2\sqrt[3]{x^2}}$ $\quad y = \frac{1}{2}x^{-2/3}$ $\quad y = \frac{1}{2}\left(-\frac{2}{3}\right)x^{-5/3}$

$y = -\frac{1}{3x^{5/3}}$

Find the slope of $f(x) = x^3 - 3x$ at the following points.

$f'(x) = 3x^2 - 3$

1. (-2, -2) $\rightarrow f'(-2) = 3(-2)^2 - 3$
$= 9$
2. (0, 0) $\rightarrow f'(0) = 3(0)^2 - 3$
$= -3$
3. (1, -2) $\rightarrow f'(1) = 3 - 3$
$= 0$

Horizontal tangent lines have a slope of 0.
Vertical tangent lines slopes are undefined

HW: p. 119 # 3 - 29 odd,
and # 31 - 42

Get organized for tomorrow's HW Quiz:

Mon: pg 82

Tues: pgs 75 & 82

Wed: pg 92

Thur: study for test

Fri: pg 103